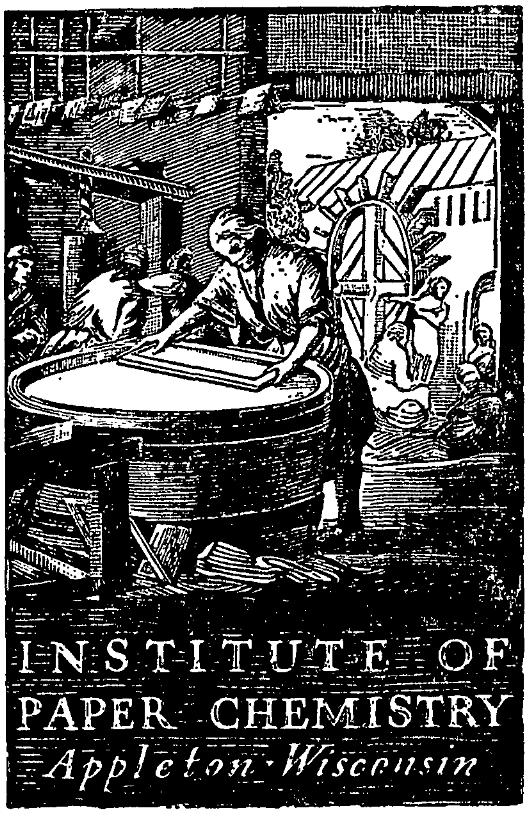

RELATIONSHIP BETWEEN SACK PERFORMANCE AND THE PROPERTIES OF SACK PAPER

PART IV. A Study of the Relationship between Uniaxial Tension Fatigue Life (Applied Strain) and the Progressive Height Sack Impact Test

Project 2033

Report Twenty-two

A Progress Report

to

MULTIWALL SHIPPING SACK PAPER MANUFACTURERS

August 31, 1962

THE INSTITUTE OF PAPER CHEMISTRY

Appleton, Wisconsin

RELATIONSHIP BETWEEN SACK PERFORMANCE AND THE PROPERTIES OF SACK PAPER
PART IV. A STUDY OF THE RELATIONSHIP BETWEEN UNIAXIAL TENSION FATIGUE LIFE (APPLIED STRAIN) AND THE PROGRESSIVE HEIGHT SACK IMPACT TEST

Project 2033

Report Twenty-Two

A Progress Report

to

MULTIWALL SHIPPING SACK PAPER MANUFACTURERS

August 31, 1962

TABLE OF CONTENTS

MEMBERS OF GROUP PROJECT 2033

Albemarle Paper Manufacturing Co.

Continental Can Company, Inc.

Crown Zellerbach Corporation

Crossett Division, Georgia-Pacific Corporation

Hudson Pulp & Paper Corp.

International Paper Company

Longview Fibre Company

Olin Mathieson Chemical Corporation, Forest Products Operations

Owens-Illinois Glass Company, Forest Products Division

St. Regis Paper Company

Scott Paper Company

Union Bag-Camp Paper Corporation

West Virginia Pulp & Paper Company

THE INSTITUTE OF PAPER CHEMISTRY

Appleton, Wisconsin

RELATIONSHIP BETWEEN SACK PERFORMANCE AND THE PROPERTIES OF SACK PAPER
PART IV. A STUDY OF THE RELATIONSHIP BETWEEN UNIAXIAL TENSION FATIGUE LIFE (APPLIED STRAIN) AND THE PROGRESSIVE HEIGHT SACK IMPACT TEST

SUMMARY

1. The repeated impact performance of a multiwall sack may be expected to be dependent in part on the tension fatigue life of the parent sack paper.

2. The materials of the second fabrication program were evaluated for Instron uniaxial fatigue life at 50, 25, and 10% R.H. (outer ply only). Each specimen was subjected to a schedule of progressively increasing applied elongation in analogy to the progressive height face impact test by which sack performance was evaluated.

3. The machine-direction fatigue life of the extensible papers was about four times that of the regular papers, on the average. The cross-direction fatigue life of the extensible papers, on the average, was about 1-1/3 times that of the regular papers.

4. Decreasing the relative humidity caused a decrease in the Instron fatigue life in each direction of the paper, in general, but the per cent decrease in either direction or in combination was not as great as that exhibited by the sacks.

5. It was noted that the variability in the Instron fatigue life was markedly lower with the progressively increasing applied elongation schedule than had been experienced earlier with a constant elongation process.

6. A number of empirical relationships between sack performance and tension fatigue life in one or both directions were studied.

7. For the combined regular and extensible papers the most precise relationships involving only one direction of the paper were obtained with machine-direction fatigue life (average difference of 13 to 18%); a separate relationship was required at each humidity, however, to account for shifts in level of sack performance with change in humidity.

8. An exploratory study was performed to determine whether the aforementioned shifts in sack performance were attributable to change in commodity behavior with humidity. A correction for commodity behavior did not fully account for the shifts in sack performance, suggesting that some factor other than parent paper fatigue life or commodity behavior influenced the sack performance when the humidity was varied.

9. Cross-direction fatigue life was moderately well correlated with sack performance in a gross sense (average difference of 20 to 33%), but the relationship lacked precision at the separate levels of humidity.

10. Multiple linear regressions involving uniaxial fatigue lives in both directions of the paper closely paralleled the simple linear regressions (average difference of 12 to 31%). That is, when compositing data at all humidities, the multiple regression was quite similar to the simple regression involving cross-direction fatigue life; at each separate humidity level, on the other hand, the multiple regressions were essentially the same as the simple correlation involving machine-direction fatigue life.

11. When the regular and extensible papers were considered separately at 50% R.H. relatively precise estimates (11 to 14%) of sack performance were achieved with fatigue life in either or both directions of the sheet.

12. In view of the above-mentioned results, it was inconclusive from this experiment whether sack performance is governed primarily by the fatigue life in one or the other direction of the parent sack paper or whether both directions are important.

13. More elaborate empirical relationships between sack performance and both uniaxial fatigue lives (involving quadratic terms or power functions) offered no improvement over the linear regressions discussed earlier.

14. Allied studies with materials from the second fabrication program have shown that the most precise relationships between sack performance and conventional paper properties at 50% R.H. were obtained with (a) machine-direction and cross-direction virgin tensile energy absorption, or (b) machine-direction and cross-direction Frag, or (c) Thwing-Albert impact fatigue.

15. Instron fatigue life offered an improvement over the aforementioned conventional paper properties with respect to prediction of sack performance at 50% R.H. as shown by the following table of average per cent difference between observed and predicted sack performance:

Paper Property	Average Difference, %
Instron fatigue life, M.D. and C. D.	11.8
Thwing-Albert impact fatigue	16.8
Tensile energy absorption, M.D. and C.D.	19.1
Frag, M.D. and C.D.	20.9

16. The increased precision in estimation of sack performance from Instron fatigue life may not justify the increase in time and skills required to evaluate the Instron fatigue life of paper samples. On the other hand, earlier studies have demonstrated that Instron fatigue life is amenable to mathematical

analysis, whereby fatigue life may be estimated from the results of simple tension tests. Perhaps even more important, experience with the effect of papermaking variables on the tension load-elongation characteristics may be translated into potential fatigue performance.

17. Instron fatigue life (as evaluated in this research study) was found to be quite highly correlated with virgin stretch, as suggested by previous analytical studies of fatigue life. Frag and Instron fatigue life did not appear to be well correlated.

18. In conclusion, it appears from this study that Instron fatigue life offers an improvement over conventional paper properties for estimation of sack performance (progressive height face drop). On the other hand, the empirical relationships developed are neither fully credible nor inclusive, indicating that much remains to be learned regarding the impact behavior of multiwall sacks.

19. Work is now in progress to evaluate the samples of the second fabrication program by means of an applied energy schedule of repeated tension. This type of stressing may bear a closer analogy to the sack impact test than the applied elongation process of the present study.

INTRODUCTION

Observation of the repeated impact performance of multiwall sacks in the laboratory indicates that the paper in the sack is subjected to repeated tension stresses. The behavior of a material in repeated stress is, by definition, a fatigue phenomenon. It seems reasonable to expect, therefore, that the repeat impact performance of a multiwall sack is governed, at least in part, by the tension fatigue life of the sack paper.

Previous studies (1-3) were concerned with (a) a theoretical and experimental survey of the behavior of plain kraft sack paper in repeated uniaxial tension and (b) a mathematical description of the mechanism of repeated tension of plain kraft paper in terms of the load-elongation curve. The latter enables estimation of uniaxial tension fatigue life from the behavior of the paper in the virgin tension test and in a single reloading test, thereby avoiding a time-consuming fatigue test. Moreover, the effect of fiber and processing variables on potential fatigue performance may be estimated, provided these effects are reflected in the simple tension tests.

The present study is concerned with determining the relationship between sack impact performance and the uniaxial fatigue life of the parent paper in each principal direction of the sheet. Ideally, this study should be preceded by the determination of two other relationships. First, inasmuch as the paper in a sack is stressed simultaneously in both principal directions during impact (4) it would be desirable to establish the relationship between sack performance and the biaxial fatigue life of the paper, the latter evaluated under conditions of stressing approximating those existing in the sack. This relationship has not been determined, however, both for lack of a biaxial fatigue test which is known to be appropriate to sack impact and for lack of a theoretical understanding of the functional relationship between paper and sack performance.

Second, it would be desirable to know the relationship between the uniaxial fatigue lives and the biaxial fatigue life of the sack paper. Experience with other materials indicates that the biaxial strength is a function of the two uniaxial strengths (in a nontrivial manner) and it may be presumed that an analogous situation exists for fatigue life. This relationship has not been established for sack paper, again primarily for lack of an adequate biaxial test method.

In spite of the aforementioned gaps in existing knowledge regarding the behavior of sack paper and the mechanics of sack impact, the present work was undertaken to study the relationship between uniaxial fatigue life and sack performance. Necessarily the mathematical relationship which is the object of this study had to be formulated empirically for the reasons mentioned above. Thus, linear or multiple regression analysis was the primary mathematical tool.

Moreover, because of inadequate understanding of the stresses and strains applied to the paper during sack impact, the method of performing the repeated tension test on the paper was largely arbitrary. It was decided to perform these tests by controlling the elongation applied to the tension specimen in each cycle. The decision to control the applied elongation rather than applied energy was influenced by several factors, namely, (a) it is believed that elongation can be controlled more effectively than energy with an Instron testing machine, (b) the mathematical description of uniaxial repeated tension (2, 3, 5) has been oriented to applied elongation, (c) experimental data are available on the magnitude of impact strains in a sack (4) but not for energy, and (d) the two types of processes appear to be equivalent for plain (or flat kraft) sack papers.

Inasmuch as the sack performance data available to this study were obtained from progressive height face impact tests on sacks from the second fabrication program (6) it was further decided to (a) progressively increase the applied elongation from cycle to cycle in the repeated tension test of the sack paper and (b) subject all samples of sack paper to the same magnitudes of strain. It is believed, therefore, that the repeated tension tests bear an analogy (though perhaps crudely) to the repeated strains which occur in progressive height sack impact.

It bears emphasizing that fatigue life is not a single-valued property for a given sample of sack paper as is, for example, the virgin stretch or tensile energy absorption or tensile strength. Fatigue life may vary from zero to an indefinitely large number for a given sample and direction, depending on the magnitude of the stress or strain applied in each cycle, and the relationship between fatigue life and applied strain is highly nonlinear (2, 3), thereby preventing simple proportioning to convert the results from one level of the input to another. Thus, it is not meaningful to speak of <u>the</u> fatigue life of a paper as though it were a unique value. While semantic considerations dictate use of the phrase "Instron uniaxial fatigue life" in numerous places in this report, it should be remembered that this actually refers to the number of safe applications which the paper sustained <u>when repeatedly strained according to a particular progressively increasing schedule of elongations</u> mentioned earlier and detailed in Test Procedure.

The scope of this study is as follows: an experimental determination of uniaxial tension fatigue life (progressively increasing elongation) was made for the materials of the second fabrication program at 50, 25, and 10% R.H. A number of correlations are studied relating sack performance to uniaxial fatigue life.

Lastly, Instron fatigue life is compared with virgin tension properties and other fatigue tests, with emphasis on their abilities to predict sack performance at 50% R.H.

MATERIALS

The materials for this study were obtained from the second fabrication program which was carried out at Union Bag-Camp Paper Corporation, Savannah, Georgia, in December, 1960 (6). This fabrication involved 26 runs of 3-ply, pasted cement sacks manufactured from 50-lb. unbleached, kraft sack paper--twelve runs of regular paper and fourteen runs of extensible paper.

Samples of the sack paper were procured at the start and end of each run of sacks during fabrication and provided the supply of materials for the evaluation of Instron fatigue life.

TEST PROCEDURE

UNIAXIAL FATIGUE LIFE TESTS

Repeated tension tests were performed by means of an Instron testing machine equipped with line clamps (7). The specimen span was six inches, the width one inch, and the deformation rate (crosshead speed) was 0.2 inch/minute for both loading and unloading.

Six machine-direction specimens and six cross-machine direction specimens were tested for each run of sacks. The tensile specimens correspond to the outer ply of the fabricated sack. Three specimens from each group of six came from the start of the production run of sacks and three from the end of the run. Each set of three specimens was comprised of one specimen from the front of the parent roll, one from the center and one from the back of the roll.

The repeated tension test was conducted by applying a prescribed elongation during each cycle of loading. That is, the crossheads of the testing machine were moved apart a prescribed distance (reckoned from the pickup of load for the cycle) and then reversed. A ten-second recovery period was observed between cycles.

The amount of elongation applied in each cycle was progressively increased from cycle to cycle, as shown in Table I. For example, each machine-direction specimen was subjected to an elongation of 0.040 inch on the first cycle, 0.050-inch elongation on the second cycle (reckoned from load pick-up), etc. Cycling was continued until the specimen broke. The number of safe cycles (i.e., not including the cycle during which rupture occurred) is, by definition, the fatigue life of the specimen.

TABLE I

SCHEDULE OF APPLIED ELONGATION IN REPEATED TENSION TESTS

Cycle Number	Applied Elongation, inch Machine Direction	Cross Direction
1	$\frac{.040}{6} \times 100 = 0.67\%$ 0.040	0.060
2	0.050	0.075
3	0.060	0.090
4	0.070	0.105
5	0.080	0.120
6	0.090	0.135
--	--	--
n	(0.040)+($\underline{n}$-1)(0.010)	(0.060)+($\underline{n}$-1)(0.015)

The schedule of applied elongations listed in Table I was arrived at from the following considerations. Progressively increasing applied elongation is believed to be somewhat analogous to the progressive height sack impact test, whereby each successive drop imparts an increasing magnitude of energy to the sack. It may be noted from Table I that the increment in applied elongation from cycle to cycle was one-quarter of the elongation of the first cycle; this is in analogy to the progressive height impact test on the sacks wherein the increment in height (six inches) is one-quarter of the initial height (twenty-four inches). Moreover, preliminary trials with machine-direction specimens revealed that this test schedule gave nonzero fatigue lives for low-stretch regular papers and not unreasonably high fatigue lives for high-stretch extensible papers.

It may be noted in Table I that for any given cycle the elongation applied to a cross-machine direction specimen was 1.5 times the elongation for a machine-direction specimen. This ratio of applied strains was selected on the basis of strain measurements performed on impacted (regular kraft) sacks ($\underline{4}$), which indicated that the strain ratio ranged from about 1.0 to 2.5; the selected ratio of 1.5 is nearly median to this range.

In the interest of economy of test time, it was found possible to test three specimens of a given type simultaneously. Inasmuch as the repeated tension test involved controlling elongation, all three specimens were subjected to the same prescribed elongation when tested simultaneously, even though their respective loads may have differed. Preliminary trials revealed that multiple specimen testing gave the same results as single specimen tests. Failure of individual specimens within a group of three was readily detectable from inspection of the composite load-elongation curve. In the main body of testing, a group of three specimens was comprised of one specimen from the front of the parent roll, one from the center and one from the back of the roll.

Fatigue life tests were performed on all runs of sack paper at 50% R.H. (room temperature). In addition, seven runs of regular paper and eight runs of extensible paper were tested at 25 and 10% R.H.

SACK IMPACT TESTS

Thirty sacks from each run were subjected to a progressive height face impact test starting at two feet and progressing by six-inch increments of height. The testing was performed as a part of the fabrication program (6).

DISCUSSION OF RESULTS

EXPERIMENTAL DATA

The Instron uniaxial fatigue lives of the sack paper and the sack fatigue lives (i.e., number of safe drops) in the progressive height face impact test at the various humidities are listed in Table II. Each entry of Instron fatigue life is the average of six determinations; sack fatigue life is the average from thirty sacks. Fatigue lives of the individual Instron specimens are listed in Tables X to XII of the Appendix.

It may be seen in Table II that, in the machine direction, the Instron fatigue lives of the extensible papers are substantially greater than those of the regular papers; for example, 13.6 versus 3.8, on the average, at 50% R.H. In general, the machine-direction fatigue life of the extensible papers was almost four times that of the regular papers. This result is to be expected because, in a repeated straining process, a high-stretch paper will outlast a low-stretch paper. In the cross direction, on the other hand, the fatigue lives of the two classes of papers are more nearly equal (6.0 versus 4.4 at 50% R.H., on the average), in keeping with their more nearly equal virgin stretches in this direction. In general, the cross-direction fatigue life of the extensible papers was about 1-1/3 times that of the regular papers.

In comparing machine direction versus cross-machine Instron fatigue lives in Table II, it should be kept in mind that the cross-direction specimens were subjected to 50% greater elongation than the machine-direction specimens on any given cycle, as discussed in Test Procedure. Thus, the observation that the in- and cross-direction fatigue lives of the regular papers are essentially equal in Table II is a reflection of the test method rather than a consequence of nearly

TABLE II

FATIGUE LIFE OF SACK PAPER AND MULTIWALL SACKS AT
VARIOUS RELATIVE HUMIDITIES

In+Cross - Code = NT

		Instron Uniaxial Fatigue Life, no. of strain applications										Sack Fatigue Life (Progressive Height Face Drop), no. of drops				
		In-Machine					Cross-Machine									
	Run	50% R.H.	25% R.H.	Diff., %[a]	10% R.H.	Diff., %[a]	50% R.H.	25% R.H.	Diff., %[a]	10% R.H.	Diff. %[a]	50% R.H.	25% R.H.	Diff., %[a]	10% R.H.	Diff., %[a]
		Regular Sack Paper														
9.25	AA	5.0					4.2					8.2				
8.0	BB	3.5	3.2	-9	3.0	-14	4.5	4.0	-11	3.0	-33	7.6	4.3	-43	1.9	-75
9.7	CC	4.0					5.7					8.4				
8.8	DD	4.0					4.8					6.6				
7.7	EE	4.0	3.0	-25	2.7	-32	3.7	3.2	-14	2.3	-38	5.2	3.4	-35	1.9	-63
7.4	FF	3.7					3.7					5.5				
8.4	GG	3.7	3.0	-19	3.0	-19	4.7	3.7	-21	3.0	-36	7.1	3.4	-52	1.0	-86
6.6	HH	2.8					3.8					6.8				
8.3	II	4.3	4.2	-2	4.0	-7	4.0	3.3	-18	2.7	-22	7.5	4.5	-40	1.7	-77
9.0	JJ	3.7	3.7	0	3.2	-14	5.3	3.2	-40	3.2	-40	9.2	4.9	-47	2.1	-77
7.5	KK	3.3	3.3	0	3.0	-9	4.2	3.7	-12	2.7	-36	6.1	3.3	-46	1.3	-79
8.6	LL	3.8	3.8	0	3.0	-21	4.8	3.8	-21	2.8	-42	6.4	2.7	-58	1.3	-80
	Av.	3.8	3.5	-8	3.1	-17	4.4	3.6	-20	2.8	-35	7.1	3.8	-46	1.6	-77
		2.2					2.0									
		Extensible Sack Paper														
16.8	MM	10.8	10.5	-3	9.3	-14	6.0	4.7	-22	3.3	-45	13.0	8.1	-38	5.2	-60
20.0	NN	14.2	12.3	-13	11.5	-19	5.8	4.2	-28	3.2	-45	14.5	7.6	-48	5.2	-64
20.6	OO	15.3	15.0	-2	14.7	-4	5.3	3.7	-30	3.0	-43	16.1	10.3	-36	7.7	-52
16.7	PP	11.5	10.8	-6	10.5	-9	5.2	4.8	-8	3.5	-33	12.7	7.8	-39	4.3	-66
20.5	QQ	14.2	13.3	-6	12.3	-13	6.3	4.7	-25	3.7	-41	15.1	7.7	-49	5.0	-67
23.1	RR	16.3	16.3	0	14.3	-12	6.8	4.8	-29	3.8	-44	17.4	9.7	-44	6.0	-65
16.3	SS	10.3					6.0					8.8				
19.4	TT	13.7	13.2	-4	11.3	-17	5.7	4.8	-16	4.0	-30	10.5	6.0	-43	4.7	-55
20.5	UU	14.7					5.8					10.7				
20.2	VV	13.2					7.0					15.1				
19.0	WW	12.8	11.8	-8	10.3	-19	6.2	5.3	-15	4.5	-27	13.0	8.8	-32	5.8	-48
18.8	XX	13.8					5.0					11.4				
20.7	YY	14.2					6.5					12.2				
21.5	ZZ	15.8					5.7					14.3				
	Av.	13.6	12.9	-5	11.8	-13	6.0	4.6	-22	3.6	-38	13.2	8.2	-41	5.5	-60
		6.0					2.0									

[a]Based on results at 50% R.H.

equal virgin stretch in the two directions. By the same token, the difference in fatigue lives between the two directions of the extensible papers is, in a sense, magnified by the particular straining schedule selected for this study.

It may be noted from Table II that decreasing the relative humidity caused a decrease in uniaxial fatigue life, in general. The effect was most pronounced in the cross direction of the papers, where decreases of 35 and 38% occurred in going from 50 to 10% R.H., on the average. The effect of humidity was much less marked in the case of the machine-direction fatigue lives; indeed, in a number of individual samples no change was noted between 50 and 25% R.H.

Inspection of the sack fatigue life results in Table II reveals that the effect of decreasing relative humidity on sack performance was substantially greater percentage-wise than it was for the Instron tests. For example, with the regular papers a decrease in relative humidity from 50 to 10% resulted in a 77% decrease in sack fatigue life, on the average, while the machine and cross-direction Instron fatigue lives decreased only 17 and 35%, respectively. While there is no known arithmetic basis for expecting the combined per cent decreases in machine and cross-machine fatigue lives to equal the per cent decrease in sack performance, it is perhaps somewhat surprising to find such differing effects in paper *versus* sacks due to relative humidity.

Regarding variability in the Instron fatigue results, inspection of the individual specimen lives in Tables X to XII of the Appendix reveals that the scatter in test results within a sample was generally modest. Typically, the fatigue lives within a sample differed by only one or two cycles. (In 93 out of 112 cases, the range was no greater than two cycles.) This magnitude of scatter is markedly lower than has been experienced with a constant elongation test schedule (3) where it was not uncommon for specimen lives to range over about 20 cycles. The lower variability of the progressive elongation test is probably attributable to the relative magnitudes of the applied elongation and the variation in virgin stretch; as the cycling continues with a given specimen, the applied elongation becomes larger and larger, overshadowing modest deviations in virgin stretch between specimens, such that all specimens of a sample fail after about the same number of cycles.

RELATIONSHIP BETWEEN UNIAXIAL FATIGUE LIVES AND SACK PERFORMANCE

The relationship between the observed fatigue lives of the sack (progressive height face drop) and of the parent sack paper in repeated tension (progressive applied elongation) was studied by means of regression analysis. The forms of the relationships investigated are empirical, in the sense that no theoretical basis exists to suggest likely functional relationships between the sack and paper properties.

Two general classes of regressions were performed: (a) simple linear regressions involving Instron fatigue life in one direction of the sack paper, and (b) multiple regressions involving linear or quadratic terms or power functions of the Instron fatigue lives in both principal directions of the paper. The multiple regressions were studied on the premise that sack performance is dependent on the biaxial fatigue life of the paper, which in turn is some function (though unknown) of the uniaxial fatigue lives in the two principal directions. Within each type of regression various partitionings of the data were explored, that is, grouping by types of paper or by levels of humidity.

All computations were performed on an IBM 1620 digital computer, using standard programs for the regression analyses. A special program was written to calculate the per cent difference between observed and estimated sack fatigue life (the latter from the regression equation) for each sample and was applied selectively to those equations that were of greatest interest.

Simple Linear Regression Analyses

The results of the simple regression analyses are presented in Table III. For example, in Regression No. 1, sack fatigue life, Z, was related to machine-direction Instron fatigue life, X, by the equation $Z = 0.603X + 2.275$ when all 56 samples of regular and extensible papers and all three humidity levels were considered. This correlation was relatively poor, having a correlation coefficient of only 0.720 and an average absolute per cent difference between observed and estimated sack life of 50.8%.

When the regular and extensible papers were considered separately (at all humidity levels) the correlations were poorer (Regressions No. 2 and 3), although there was some improvement in the per cent difference with the extensible samples. On the other hand, partitioning the data by humidity level resulted in a marked improvement in the correlations (Regression Numbers 4, 5, and 6) with correlation coefficients ranging from 0.899 to 0.960 and average differences of 13.3 to 18.5%.

These results are shown graphically in Fig. 1 and 2 which are graphs of sack fatigue life *versus* machine-direction Instron fatigue life. Clearly, straight lines do not adequately describe the data in their entirety or when they are partitioned by types of paper (Fig. 1). But when grouped by humidity level (Fig. 2) each group of data clusters more nearly about one of three straight lines (Regressions 4-6). In a crude sense, the slopes of these latter three lines are about equal (namely, 0.636, 0.463, and 0.443) and differ mainly in their intercepts. This observation indicates that sack life is reasonably

TABLE III

SIMPLE LINEAR REGRESSIONS OF SACK FATIGUE LIFE ON INSTRON FATIGUE LIFE

Independent Variable	Regression No.	Samples Included	Relative Humidity, %	No. of Observations	Correlation Coefficient	Equation[a]	Average Absolute Difference, %[b]
M.D. fatigue life, $\underline{X}$	1	Reg. & Ext.	50, 25, 10	56	0.720	$\underline{Z} = 0.603\underline{X} + 2.275$	50.8
	2	Reg.	50, 25, 10	26	0.575	$\underline{Z} = 2.640\underline{X} - 4.627$	56.0
	3	Ext.	50, 25, 10	30	0.572	$\underline{Z} = 1.112\underline{X} - 4.569$	31.0
	4	Reg. & Ext.	50	26	0.899	$\underline{Z} = 0.636\underline{X} + 4.573$	13.3
	5	Reg. & Ext.	25	15	0.924	$\underline{Z} = 0.463\underline{X} + 2.237$	14.1
	6	Reg. & Ext.	10	15	0.960	$\underline{Z} = 0.443\underline{X} + 0.245$	18.5
	7	Reg.	50	12	0.271	$\underline{Z} = 0.601\underline{X} + 4.755$	13.1
	8	Ext.	50	14	0.583	$\underline{Z} = 0.779\underline{X} + 2.585$	13.7
C.D. fatigue life, $\underline{Y}$	9	Reg. & Ext.	50, 25, 10	56	0.877	$\underline{Z} = 3.127\underline{Y} - 6.364$	29.2
	10	Reg.	50, 25, 10	26	0.867	$\underline{Z} = 2.594\underline{Y} - 5.073$	33.4
	11	Ext.	50, 25, 10	30	0.812	$\underline{Z} = 2.769\underline{Y} - 3.957$	20.0
	12	Reg. & Ext.	50	26	0.808	$\underline{Z} = 3.054\underline{Y} - 5.696$	17.1
	13	Reg. & Ext.	25	15	0.647	$\underline{Z} = 2.423\underline{Y} - 3.831$	27.5
	14	Reg. & Ext.	10	15	0.624	$\underline{Z} = 2.347\underline{Y} - 3.946$	50.9
	15	Reg.	50	12	0.672	$\underline{Z} = 1.260\underline{Y} + 1.445$	11.1
	16	Ext.	50	14	0.344	$\underline{Z} = 1.413\underline{Y} + 4.795$	14.1
$\left\{ \begin{array}{l} \text{M.D. for regular} \\ \text{C.D. for extensible} \end{array} \right.$	17	Reg. & Ext.	50, 25, 10	56	0.850	$\underline{Z} = 3.065\frac{\underline{X}}{\underline{Y}} - 5.757$ $\underline{X}$ for Regular $\underline{Y}$ for Extensible	38.1
$\left\{ \begin{array}{l} \text{C.D. for regular} \\ \text{M.D. for extensible} \end{array} \right.$	18	Reg. & Ext.	50, 25, 10	56	0.743	$\underline{Z} = 0.634\frac{\underline{Y}}{\underline{X}} + 1.943$ $\underline{Y}$ for Regular $\underline{X}$ for Extensible	47.1

[a]Symbols: $\underline{Z}$ = Sack fatigue life (progressive height face drop); $\underline{X}$ = M.D. Instron fatigue life; $\underline{Y}$ = C.D. Instron fatigue life.

[b]Based on observed sack life.

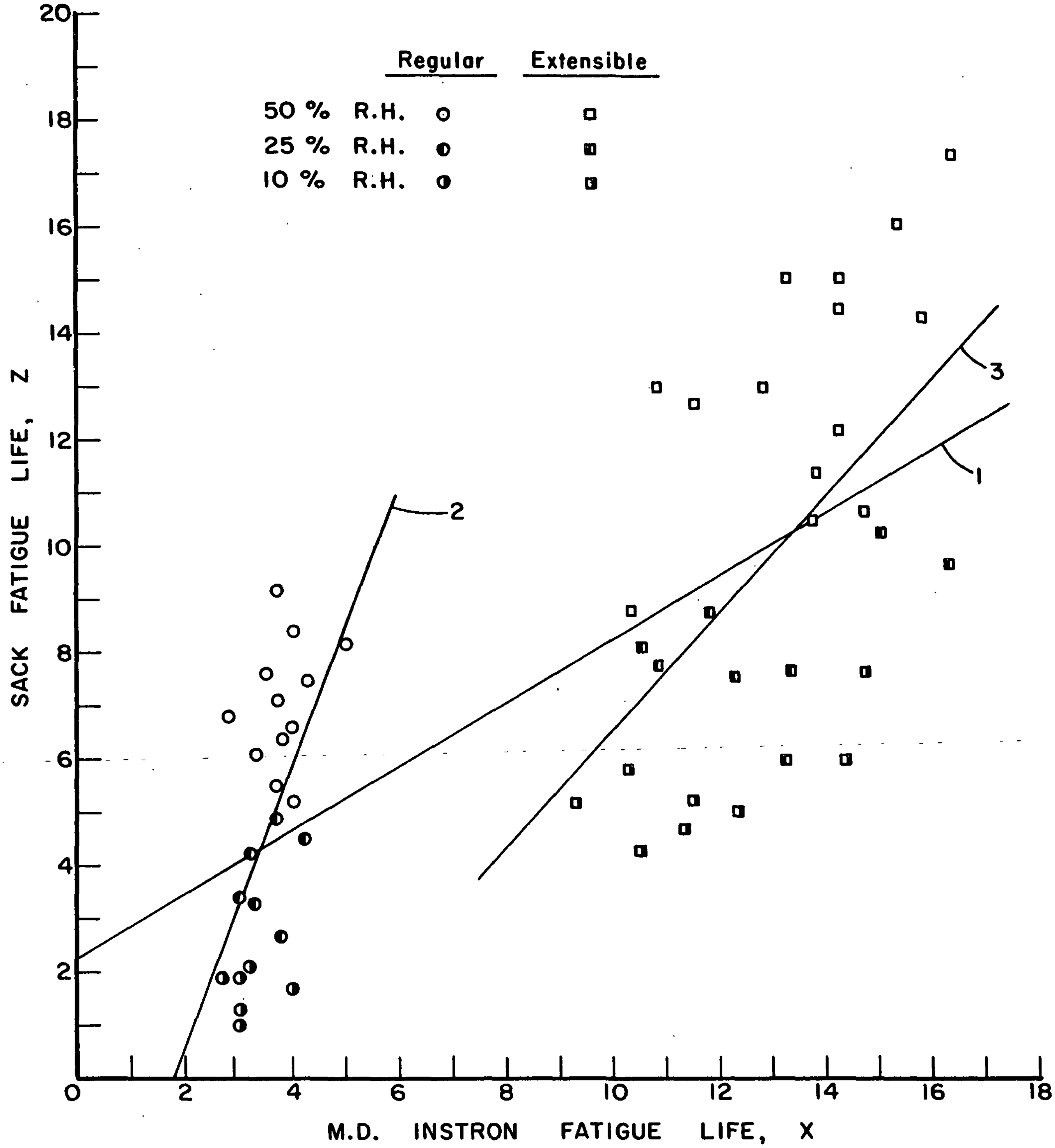

Figure 1. Relationship Between Sack Fatigue Life (Progressive Height Face Impact) and Machine-Direction Instron Uniaxial Fatigue Life (Regressions 1 to 3)

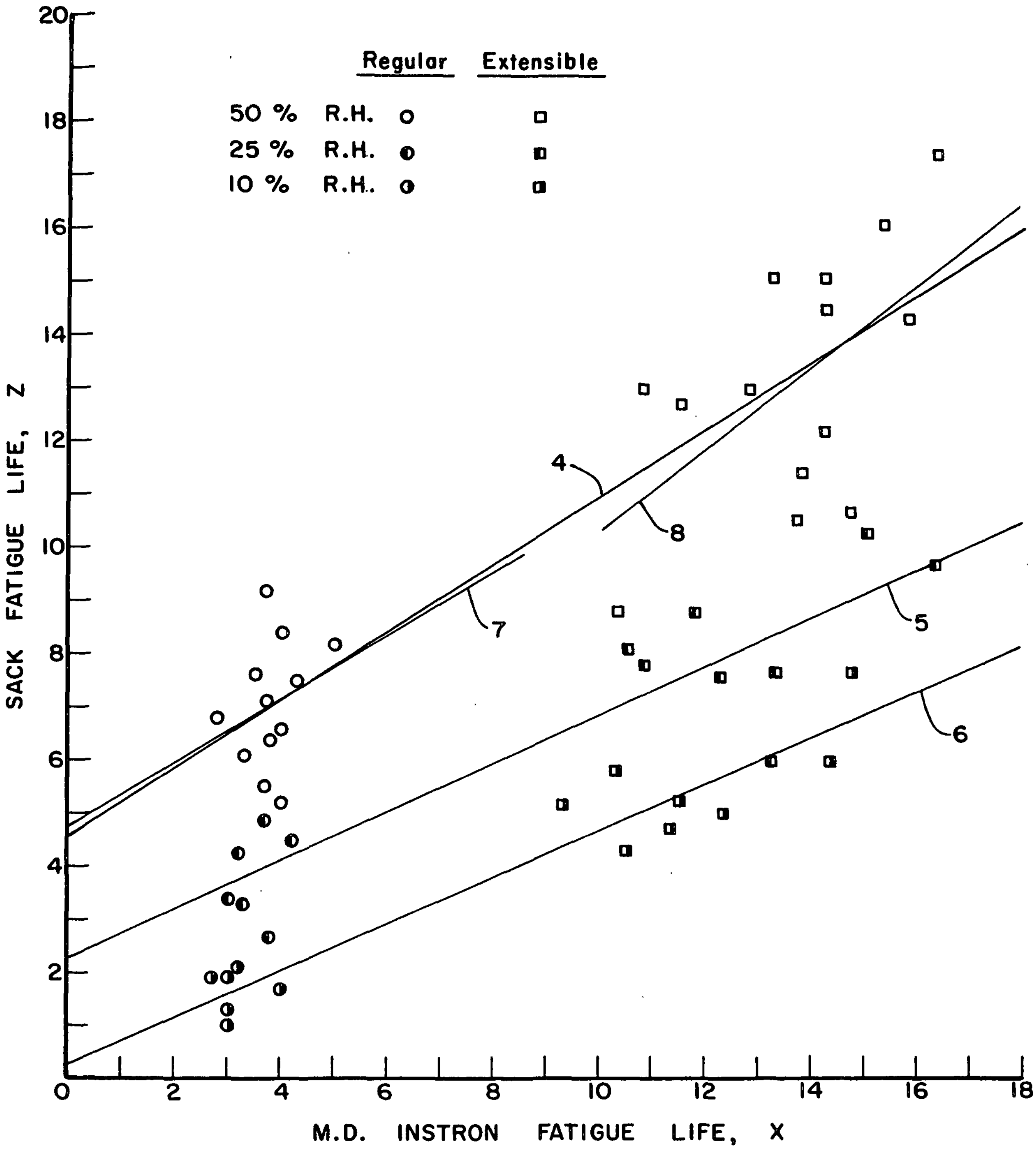

Figure 2. Relationship Between Sack Fatigue Life (Progressive Height Face Impact) and Machine-Direction Instron Uniaxial Fatigue Life (Regressions 4 to 8)

well described in terms of the machine-direction fatigue life but that some other factor(s) is operative as the humidity is varied which is not reflected in the machine-direction Instron fatigue test. Likely possibilities for the other factor are considered to be (a) cross-direction fatigue life, (b) effect of moisture on commodity, or (c) effect of moisture on crease strength.

A further subdivision of the 50% R.H. data was studied, namely, the regular papers and extensible papers separately (Regressions 7 and 8). Inspection of the regression equations in Table III or the graph of Fig. 2 reveals that the relationship for either type of paper taken separately does not differ appreciably from the combined Regression 4. This observation indicates that a single relationship applies for both regular and extensible papers--a desirable circumstance. (A subdivision of the data between regular and extensible papers at 25 and 10% R.H. was not studied because of the small number of observations.)

Regressions 9 through 16 pertain to the simple linear correlation between sack life and cross-direction Instron fatigue life. Unlike the machine-direction results described above, the cross-direction fatigue life was moderately well correlated with sack life over all samples (correlation coefficient = 0.877, average difference = 29.2%) and within a type of paper. These results are shown graphically in Fig. 3. Partitioning by humidity, on the other hand, led to a marked worsening of the correlation coefficients and of the average per cent difference at 10% relative humidity (see Regression 12 to 14). A graphical presentation of these latter results is given in Fig. 4. It appears from these results that while cross-direction fatigue life is reasonably well correlated with sack performance in a gross sense, the relationship becomes less precise than the analogous relationship involving machine-direction fatigue life when the data are segregated by humidity level.

When the 50% R.H. data are further subdivided according to type of paper (Regressions 15 and 16), an improvement is made in the per cent difference relative to the combined Regression 12 for both papers (although this improvement is not reflected in the correlation coefficient). It may be noted from the regression equations or Fig. 4 that the slopes of the individual regressions (15 and 16) are nearly equal, but quite different from the combined Regression 12. This suggests that, while the cross-direction fatigue life of either regular or extensible paper is well related to sack performance at 50% R.H., some other factor may be operative between classes of paper which is not reflected in cross-direction fatigue life.

Regression No. 17 was performed with machine-direction fatigue life for the regular papers and cross-direction fatigue life for the extensible papers and extending over all samples and humidities. This selection of paper properties may be termed the "weaker direction" analysis in the sense that for either type of paper the direction was selected for which the fatigue life was generally lower than in the other direction [the same choice of directions would have been made on the basis of virgin stretch or tensile energy absorption (6)]. The correlation coefficient of this regression was 0.850 and the average difference was 38.1%, indicating that no over-all improvement was obtained through this partitioning of the data.

Conversely, Regression No. 18 was performed with the "stronger direction" properties, that is, cross-direction fatigue life for regular papers and machine-direction fatigue life for extensible papers. This partitioning resulted in an even lower correlation coefficient than in the previous case, namely 0.743 versus 0.850, and a higher per cent difference, namely, 47.1 versus 38.1%.

Reviewing the results of the above-mentioned simple regression analyses of the combined regular and extensible papers, it appears that sack fatigue life was moderately well related to one or the other uniaxial fatigue lives of the parent

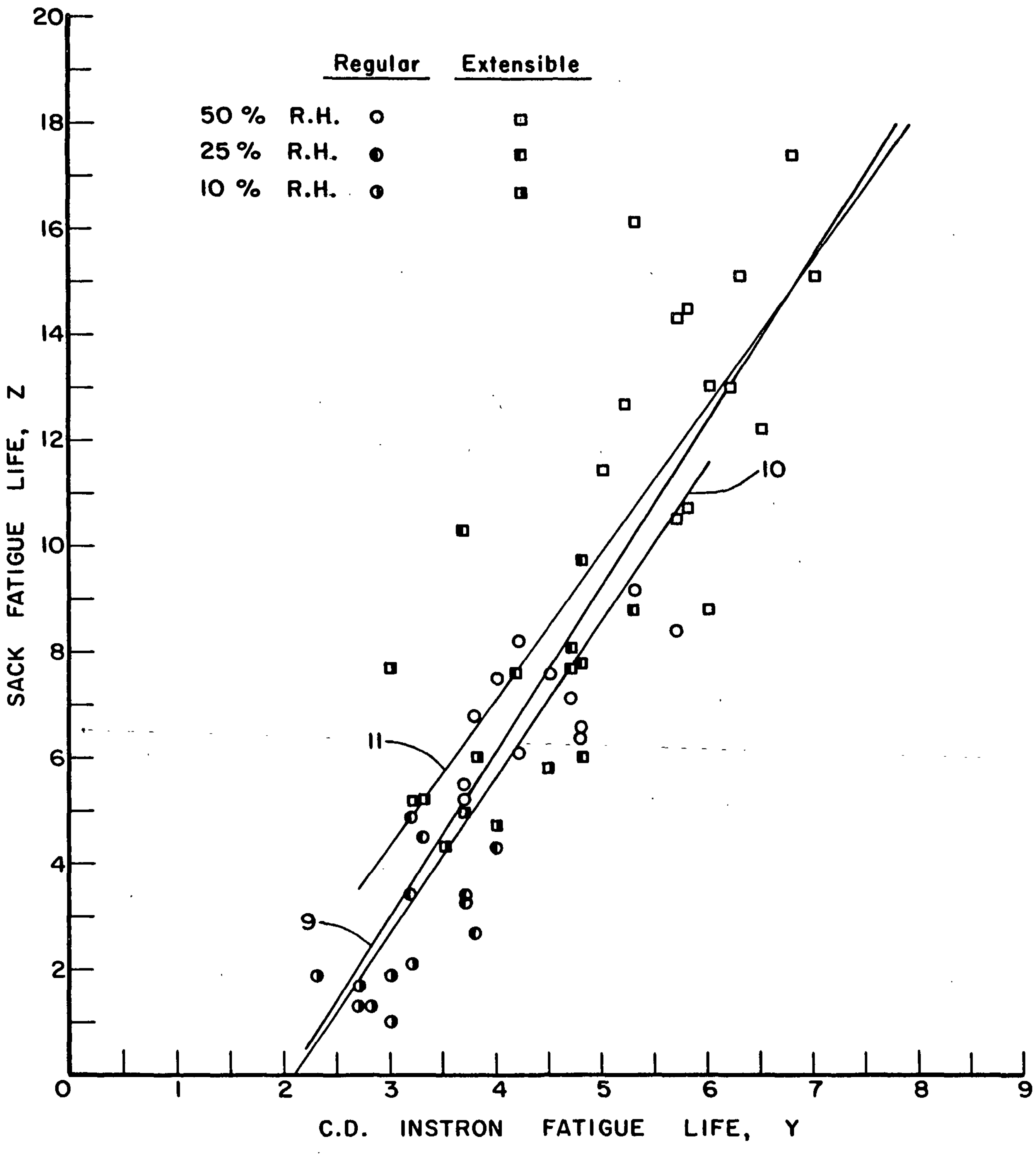

Figure 3. Relationship Between Sack Fatigue Life (Progressive Height Face Impact) and Cross-Direction Instron Uniaxial Fatigue Life (Regressions 9 to 11)

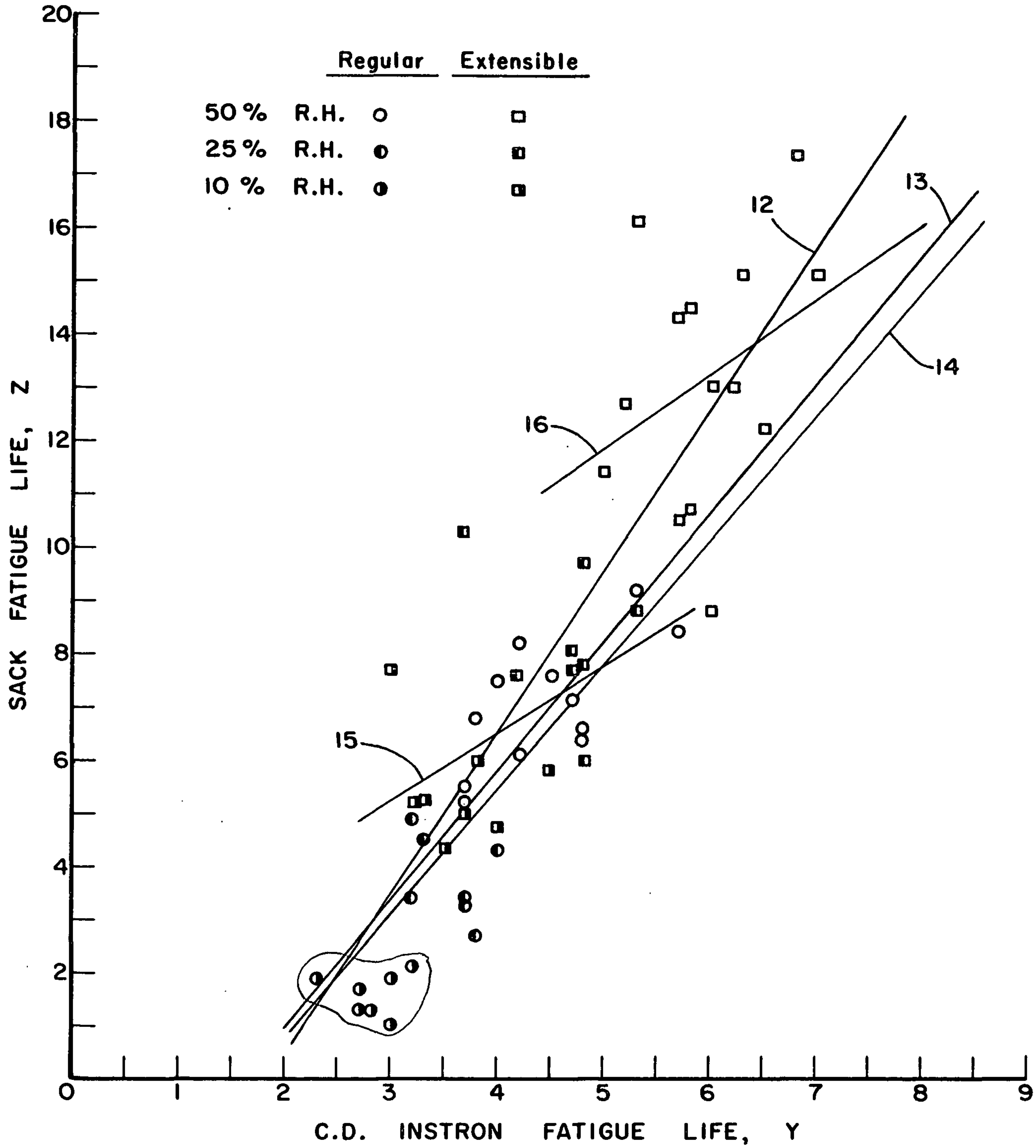

Figure 4. Relationship Between Sack Fatigue Life (Progressive Height Face Impact) and Cross-Direction Instron Uniaxial Fatigue Life (Regressions 12 to 16)

paper. In general, the most precise relationships involved machine-direction fatigue life; three separate equations were required, however, to account for shifts in level of sack performance with change in humidity--shifts that were not accounted for by the machine-direction fatigue life of the paper. Cross-direction fatigue life was reasonably well correlated with sack performance in a gross sense but the relationship lacked precision at the separate humidities. Relatively high precision of the estimate (11 to 14%) was achieved with either direction of the paper when regular and extensible papers were considered separately at 50% R.H.

Multiple Regression Analyses

The results of the multiple regression analyses are listed in Table IV. When the machine-direction and cross-direction fatigue lives were combined linearly over all samples and humidities (Regression No. 19), their relationship to sack performance was given by the equation

$$Z = 0.269X + 2.467Y - 5.757 \tag{1}$$

The correlation coefficient was 0.916 which is higher than when either uniaxial fatigue life was considered separately (viz. 0.720 and 0.877 for Regressions No. 1 and 9). Moreover, the average per cent difference between observed and estimated sack life was 24.5%, which is an improvement over the predictive ability of either uniaxial fatigue life by itself (namely, 50.8 and 29.2%). It appears, therefore, that combining the two uniaxial fatigue lives offered a modest improvement in the empirically formulated relationship between sack performance and parent paper fatigue lives.

Figure 5 is a graph showing the agreement between observed sack fatigue life and the estimated fatigue life calculated from Equation (1) (Regression 19). The straight lines constructed on the graph indicate the magnitude of the per cent

TABLE IV

MULTIPLE REGRESSIONS OF SACK FATIGUE LIFE ON INSTRON FATIGUE LIVES

Description of Regression[a]	Regression No.	Samples Included	Relative Humidity, %	No. of Observations	Correlation Coefficient	Equation[a]	Average Absolute Difference, %[b]
Linear ($\underline{Z} = \underline{aX} + \underline{bY} + \underline{c}$)	19	Reg. & Ext.	50, 25, 10	56	0.916	$Z = 0.269X + 2.467Y - 5.757$	42.5
	20	Reg.	50, 25, 10	26	0.888	$\overline{Z} = 1.013\overline{X} + 2.288\overline{Y} - 7.502$	30.6
	21	Ext.	50, 25, 10	30	0.881	$\underline{Z} = 0.697\underline{X} + 2.398\underline{Y} - 11.132$	17.2
	22	Reg. & Ext.	50	26	0.916	$Z = 0.483X + 1.055Y + 0.423$	11.8
	23	Reg. & Ext.	25	15	0.925	$\overline{Z} = 0.481\overline{X} - 0.187\overline{Y} + 2.855$	14.1
	24	Reg. & Ext.	10	15	0.960	$\underline{Z} = 0.445\underline{X} - 0.032\underline{Y} + 0.330$	18.5
	25	Reg.	50	12	0.704	$Z = 0.459X + 1.222Y - 0.138$	11.0
	26	Ext.	50	14	0.650	$\overline{Z} = 0.740\overline{X} + 1.183\overline{Y} - 3.921$	12.4
Quadratic ($\underline{Z} = \underline{aX^2} + \underline{bXY} + \underline{cY^2} + \underline{d}$)	27	Reg. & Ext.	50, 25, 10	56	0.923	$Z = 0.0372X^2 - 0.0878XY + 0.359Y^2 - 0.0508$	24.6
	28	Reg.	50, 25, 10	26	0.881	$\overline{Z} = 0.0297\overline{X}^2 + 0.262\overline{XY} + 0.159\overline{Y}^2 - 1.590$	
	29	Ext.	50, 25, 10	30	0.887	$\underline{Z} = 0.342\underline{X}^2 - 0.0400\underline{XY} + 0.296\underline{Y}^2 - 1.131$	
	30	Reg. & Ext.	50	26	0.923	$Z = 0.0386X^2 - 0.0459XY + 0.154Y^2 + 4.150$	11.8
	31	Reg. & Ext.	25	15	0.912	$\overline{Z} = 0.0078\overline{X}^2 - 0.0791\overline{XY} - 0.0819\overline{Y}^2 + 3.893$	
	32	Reg. & Ext.	10	15	0.952	$\underline{Z} = 0.0229\underline{X}^2 - 0.0124\underline{XY} + 0.0428\underline{Y}^2 + 1.036$	
Product of power functions ($\underline{Z} = \underline{cX^aY^b}$)	33	Reg. & Ext.	50, 25, 10	56	0.894	$Z = 0.269X^{0.311}Y^{1.741}$	25.0
	34	Reg.	50, 25, 10	26	0.862	$\overline{Z} = 0.0776\overline{X}^{0.828}\overline{Y}^{2.216}$	
	35	Ext.	50, 25, 10	30	0.876	$\underline{Z} = 0.113\underline{X}^{0.970}\underline{Y}^{1.211}$	
	36	Reg. & Ext.	50	26	0.922	$Z = 1.680X^{0.347}Y^{0.644}$	11.9
	37	Reg. & Ext.	25	15	0.925	$\overline{Z} = 2.430\overline{X}^{0.645}\overline{Y}^{-0.287}$	
	38	Reg. & Ext.	10	15	0.952	$\underline{Z} = 0.602\underline{X}^{0.951}\underline{Y}^{-0.117}$	

[a]Symbols: $\underline{Z}$ = Sack fatigue life (progressive height face drop); $\underline{X}$ = M.D. Instron fatigue life; $\underline{Y}$ = C.D. Instron fatigue life.
[b]Based on observed sack life.

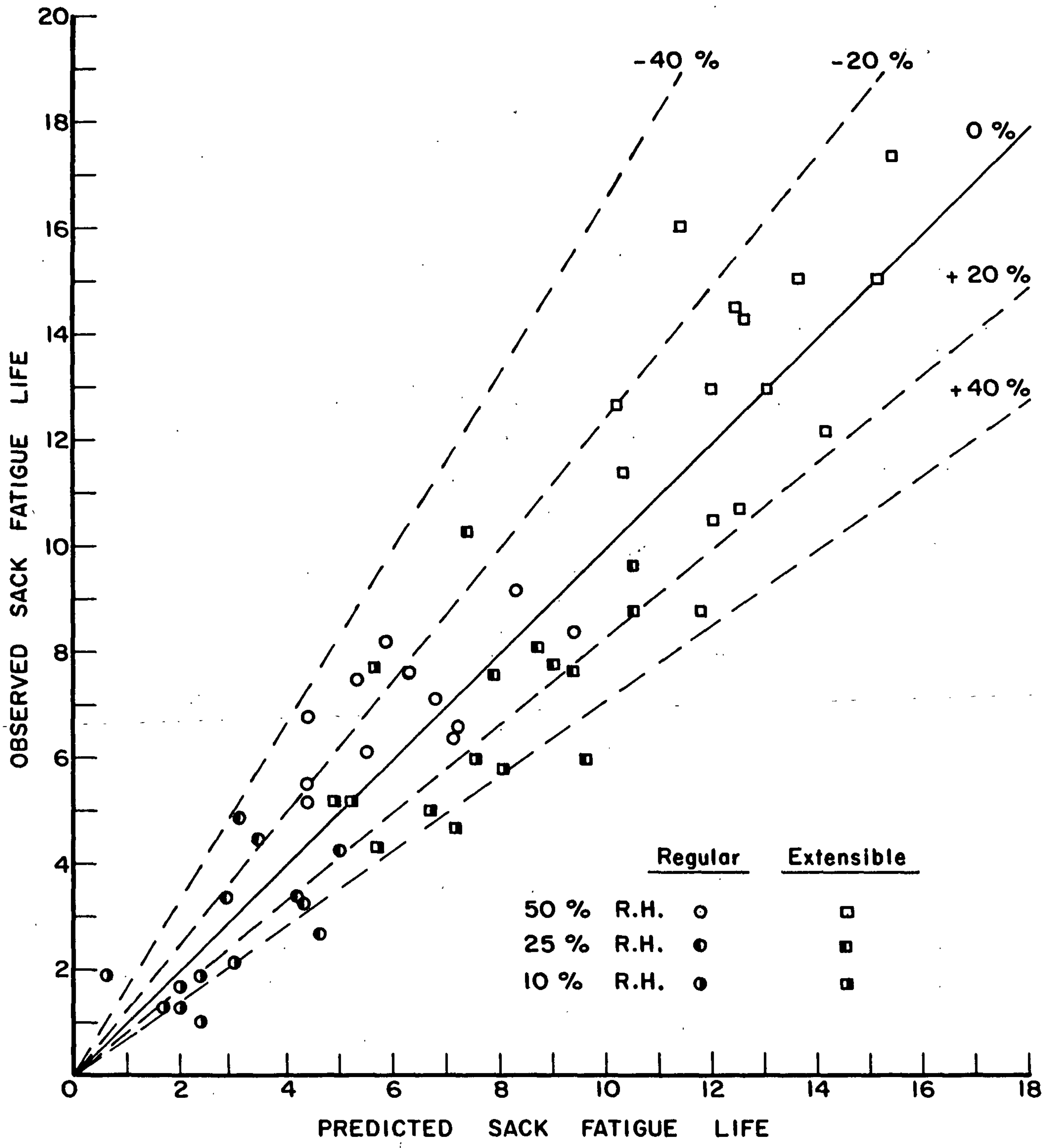

Figure 5. Comparison of Observed and Predicted Sack Fatigue Life (Based on (Regression No. 19)

difference between observed and estimated sack fatigue life. It may be seen that the difference between observed and estimated sack life was less than ± 20% in 30 of the 56 samples (i.e., in 54% of the samples studied). Forty-nine of the 56 estimates were within ± 40% of the observed sack lives. The average difference was 24.5%, as noted earlier.

Caution should be exercised in inferring the relative importance of machine-direction and cross-direction fatigue life from the multiplying coefficients of Equation (1). The observation that the coefficient of the cross-direction fatigue life is about ten times greater than the machine-direction coefficient in part reflects the differing magnitudes of the fatigue lives in the two directions. For example, with the extensible papers (which probably strongly influence the regression constants because of the wide range of extensible properties in this sampling program) the machine-direction fatigue lives are generally two to three times greater than the cross-direction fatigue lives. Thus, the multiplying coefficients are in part scaling factors to account for the differing magnitudes of fatigue life in the two directions, which in turn are influenced by the method of performing the fatigue test.

Moreover, it is suggested that evaluation of the relative importance of each direction of the paper should also account for the magnitude of the likely variation in either fatigue life. While it is true that one unit change in cross-direction fatigue life results in 10 times greater change in sack life than does one unit change in machine-direction life [according to Equation (1)], it is apparent from the data of Table II that cross-direction fatigue life is less prone to variation in commercial samples than is machine-direction fatigue life.

Regressions 20 and 21 pertain to the regular and extensible papers taken separately (at all three humidities). This grouping of the data resulted in a slight worsening of the correlation coefficients relative to Regression No. 19, although there was a marked improvement in the per cent difference in the case of the extensible papers, namely, 17.2%, on the average.

Partitioning the data by humidity level (Regressions 22 through 24) resulted in substantial increases in correlation coefficient (0.916, 0.925, and 0.960 at 50, 25, and 10% relative humidity, respectively) and more favorable average per cent differences (11.8, 14.1, and 18.5%, respectively). It may be noted that these regressions parallel quite closely the analogous simple linear regressions involving machine-direction Instron fatigue life. The correlation coefficients and the average per cent differences are nearly equal at each of the three humidity levels. Moreover, the coefficients of the machine-direction fatigue life (symbol $\underline{X}$) in the multiple regression equations parallel to some extent those of simple regressions involving machine-direction fatigue life.

It should also be noted that in the equations of Regressions 22 through 24, the coefficient of cross-direction fatigue life is highly "unstable" between the three humidity levels. At 25 and 10% relative humidity, the coefficient assumes negative values, indicating that an increase in cross-direction fatigue life tends to decrease sack life--a trend which seems intuitively unreasonable. It may be that the range of cross-direction fatigue lives presented by the samples of this study was such that a reliable estimate of the cross-direction coefficient was not possible when each humidity is considered separately. On the other hand, it may be reasoned that the negative coefficients of the cross-direction terms ($\underline{Y}$) in Regressions 23 and 24 are not significantly different from zero. Noting further that the coefficient of $\underline{Y}$ in the Regression 22 (50% R.H.) has decreased substantially from its value in Regression 19, it may be argued that the cross-direction fatigue life is relatively unimportant to the relationship between sack life and paper properties at each humidity level and that the multiple Regressions 22 through 24 are not really an improvement over the simple linear regressions involving solely machine-direction fatigue life (Regressions 4 to 6).

Lastly, the data at 50% R.H. was subdivided between regular and extensible papers, leading to Regressions 25 and 26. Although the correlation coefficients were markedly lower than with the combined data at 50% R.H. (Regression 22), the regression equations and the average differences were quite comparable to Regression 22.

Reviewing the above comparisons between the simple and multiple linear regressions, there appears to be a rather enigmatic situation. If the data are taken in their entirety, a reasonably good multiple correlation is obtained between sack life and the two uniaxial fatigue lives (Regression 19). Consideration of average per cent differences indicates that nearly as precise a relationship is obtained with cross-direction fatigue life alone (that is, omitting machine-direction fatigue life) (see Regression 9). On the other hand, when the data are viewed at each of the three humidity levels separately, substantially more precise relationships with sack life are obtained (Regressions 22 through 24) and it appears that the relationships are virtually unimpaired if machine-direction fatigue life alone is used (that is, cross-direction fatigue life omitted) (Regressions 4 through 6). Thus, it is inconclusive from these experimental data whether one or the other directions of the sack paper dominates in sack performance or whether both directions are important--the inferences which may be drawn depend on the grouping of the data.

Returning to consideration of the multiple regression analyses summarized in Table IV, Regressions 27 to 32 and Regressions 33 to 38 were performed with quadratic terms and products of power functions in the machine-direction and cross-direction Instron fatigue lives. The quadratic expression was selected for study as being somewhat more general than the linear equation; in particular, the XY term permits an interaction between the machine-direction and cross-direction fatigue lives and thereby introduces (although perhaps crudely) the interaction concept frequently associated with the behavior of materials under biaxial stress. The product of power functions was also selected for its generality.

It may be seen in Table IV that both the quadratic expression and the power function equation yielded results virtually identical with the multiple linear regression. That is, for each grouping of the data, the correlation coefficients and the average per cent differences are essentially the same in each of the three types of equations. It may be noted that even the instability of the empirical constants associated with cross-direction fatigue life is evident in the quadratic and power function analyses. Thus, no further insight into the sack *versus* paper relationship is afforded by these last considered empirical equations.

Effect of Humidity on Commodity Behavior

The observed shifts in level of sack performance with change in humidity which are not accounted for by machine-direction fatigue life alone, as discussed earlier, apparently are not explained by including cross-direction fatigue life. It may be of interest, therefore, to explore the possibility that the shifts are attributable to the effect of humidity on the behavior of the commodity, that is, the cement. The progressive height sack impact test was repeated on four runs of sacks (GG, LL, NN, and OO) at 50 and 10% relative humidity with 1-mil polyethylene inserts lining the sacks. Cement conditioned at 50% R.H. was used in the sack impact tests performed at both 50 and 10% R.H. The plastic inserts were slightly larger than the sacks and thus offered a minimum of additional strength to the sack. It may be anticipated, therefore, that because of the polyethylene inserts, the impact behavior of the cement was the same at 10% as at 50% R.H.

The results of this exploratory study are presented in Table V. Also shown are the sack performance data without plastic inserts extracted from Table II. It may be seen from the first two data columns in Table V that the "constant" commodity markedly increased the sack performance at 10% R.H. It may be reasoned that the higher moisture content of the cement in the sacks with plastic inserts permitted less flow of the cement and, therefore, imposed a less severe impact to the paper of the sack. This reasoning is confounded, however, by noting from Table V that at 50% R.H. three of the sack samples with inserts tested appreciably higher than the sacks without inserts, in contradiction to the assumption that the behavior of the cement at 50% R.H. was the same both with and without the inserts. This discrepancy is possibly due to (a) a strength contribution from the plastic insert and/or (b) the insert interfered with the normal flow of the cement within the multiwall sack.

An attempt is made in Table V to correct for the possible effect of the plastic insert. For this purpose the 10% R.H. fatigue life from a sack with insert is diminished in proportion to the ratio of the without-to-with insert sack lives at 50% R.H. For example, for Run NN

$$6.9 = \frac{14.5}{17.1} \times 8.1 \ .$$

These corrected values of sack life are given in the last column of Table V.

Figure 6 is a graph to facilitate answering the underlying question of whether or not the apparent shift in sack performance with relative humidity can be attributed to a variation in behavior of the cement. The two solid lines are identical with the 50 and 10% R.H. curves shown earlier in Fig. 2 (Regressions 4 and 6). The plotted points associated with these curves for Runs GG, LL, NN, and OO are shown on the graph. The dashed line and associated points pertain to the data from Table V for plastic inserts corrected as explained earlier (the line was fitted visually). Thus, the 50% R.H. solid line and the 10% R.H. dashed line may be interpreted as the relationships between sack life and machine-direction Instron fatigue life when the behavior of the commodity is held constant in so far as moisture effects are concerned.

TABLE V

COMPARISON OF SACK PERFORMANCE WITH AND WITHOUT PLASTIC INSERTS

Run	Type of Paper	Sack Fatigue Life: Without Plastic Insert	With Plastic Insert	With Plastic Insert--Corrected[a]
		50% R.H.		
GG	Regular	7.1	7.0	7.1
LL	Regular	6.4	7.1	6.4
NN	Extensible	14.5	17.1	14.5
OO	Extensible	16.1	21.6	16.1
		10% R.H.		
GG	Regular	1.0	3.6	3.6
LL	Regular	1.3	3.3	3.0
NN	Extensible	5.2	8.1	6.9
OO	Extensible	7.7	10.2	7.6

[a]Corrected for possible restraint or strengthening by plastic insert, as explained in text.

It is seen that there is still considerable shift in level of sack performance with humidity even after accounting for commodity effects. Thus, the results of this exploratory experiment indicate that sack performance varied with relative humidity in a manner which could not be fully accounted for by the accompanying changes in machine-direction fatigue life or in the behavior of the commodity.

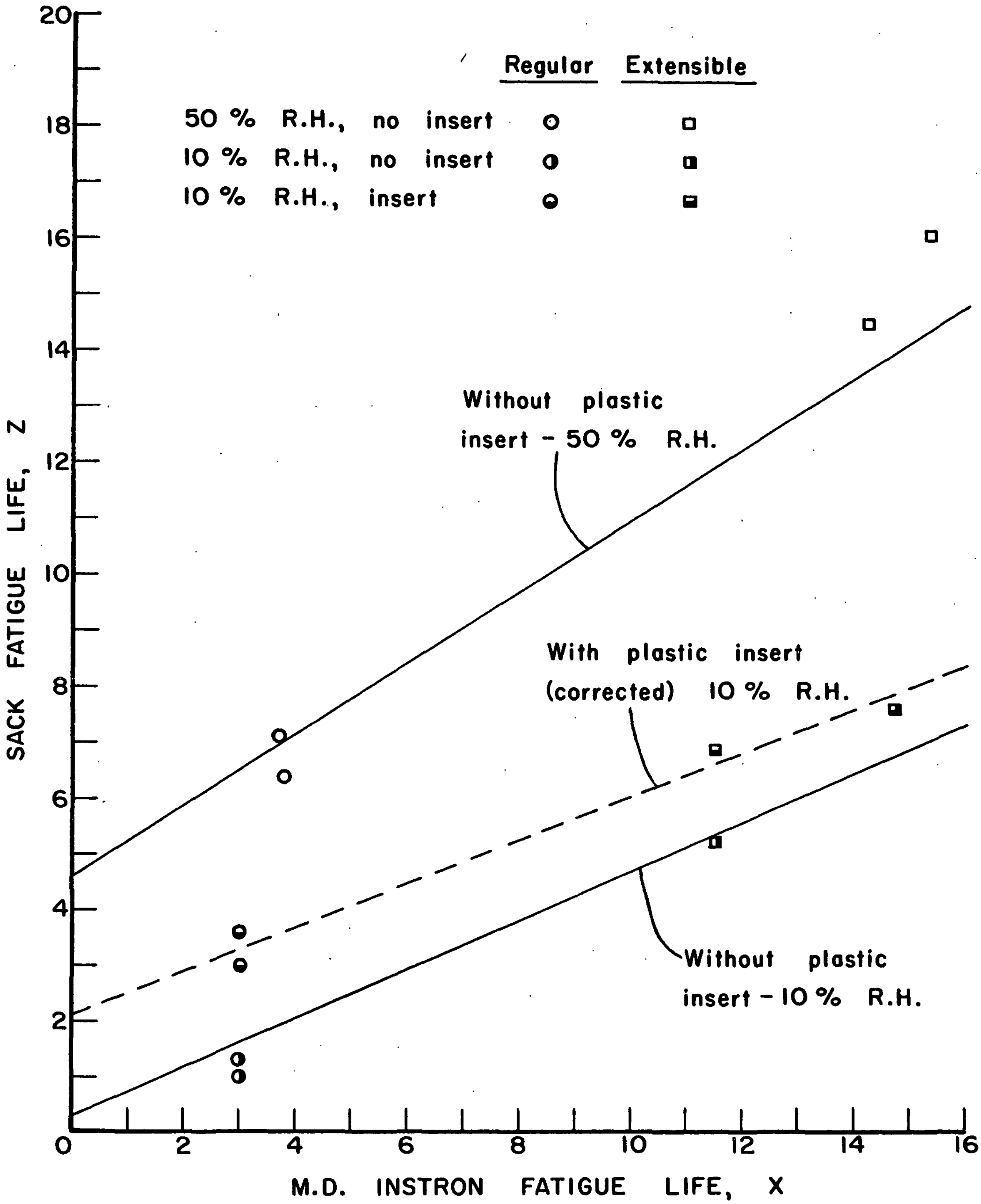

Figure 6. Relationship Between Sack Fatigue Life and Machine-Direction Instron Fatigue Life at 50 and 10% R.H. with Behavior of Commodity Held "Constant"

It has been noted during sack impact tests that the incidence of crease failures increases as the relative humidity is decreased from 50 to 10% relative humidity. It may be appropriate, therefore, to direct future work to explore the possibility that crease strength may account for the shift in level of sack performance with change in relative humidity.

COMPARISON OF INSTRON FATIGUE LIFE AND OTHER PAPER PROPERTIES

It may be of interest to examine the correlation between Instron fatigue life and other properties of the sack paper derived from simple tension tests or conventional fatigue testing machines. It is believed, however, that better perspective is afforded by first comparing the several types of properties with respect to their ability to predict sack performance. This comparison may be studied most effectively in terms of the sack performance at 50% R.H., for which the results of the conventional paper tests have been reported in Reference (6).

Table VI summarizes the results of the regression analyses involving Instron fatigue life, tensile energy absorption, Frag and Thwing-Albert impact fatigue. The regression equations containing Instron fatigue lives are multiple Regressions No. 22, 25, and 26 presented previously in Table IV. The regressions involving the remaining properties are taken from Tables XX and XXII of Reference (6) and were regarded in that report as the most favorable relationships between progressive height sack impact performance and conventional paper properties. The regressions are summarized in terms of the regression equation, the correlation coefficient, the average per cent difference between predicted and observed sack performance, and the distribution of per cent differences.

It may be seen in Table VI that with the combined regular and extensible samples the correlation coefficients are comparably high for all types of properties, although a slightly higher coefficient was obtained with Thwing-Albert impact

TABLE VI

COMPARISON OF RELATIONSHIPS BETWEEN SACK PERFORMANCE (PROGRESSIVE HEIGHT FACE DROP) AND VARIOUS PAPER PROPERTIES

(50% Relative humidity)

Paper Property	Equation[a]	Sample	Correlation Coefficient	Average Difference, %[b]	Per Cent of Comparisons Within ± 15	Per Cent of Comparisons Within ± 25
Instron fatigue life, M.D. and C.D.	$Z = 0.483X + 1.055Y + 0.423$	Reg. & Ext.	0.92	11.8	73	92
	$Z = 0.459X + 1.222Y - 0.138$	Reg.	0.70	11.0	83	100
	$Z = 0.740X + 1.183Y - 3.921$	Ext.	0.65	12.4	57	93
Thwing-Albert impact fatigue	$Z' = 13.415T_a + 114.0$	Reg. & Ext.	0.94	16.8	50	73
	$Z' = 7.78T_a + 201.5$	Reg.	0.69	15.2	58	75
	$Z' = 14.6T_a + 50.7$	Ext.	0.82	15.1	57	71
Tensile energy absorption, M.D. and C.D.	$Z' = 430W_x + 1169W_y - 361.6$	Reg. & Ext.	0.92	19.1	62	77
	$Z' = 781W_x + 555W_y - 190.8$	Reg.	0.84	12.1	67	92
	$Z' = 385W_x + 1676W_y - 599.4$	Ext.	0.73	17.9	64	71
Frag, M.D. and C.D.	$Z' = 1.534F_x - 0.204F_y - 144.7$	Reg. & Ext.	0.92	20.9	50	69

[a] Symbols: Z = sack fatigue life; Z' = safe inches of drop; X = M.D. Instron fatigue life; Y = C.D. Instron fatigue life; T_a = T.A. impact fatigue; F_x = Frag burst energy, M.D.; F_y = Frag burst energy, C.D.

[b] Based on observed sack performance.

fatigue (0.94 versus 0.92 for the other tests). On the basis of per cent differences between observed and predicted sack performance, however, the relationship involving Instron fatigue lives was the most precise, incurring an average difference of 11.8%. The Thwing-Albert impact fatigue equation was the next most precise, with an average difference of 16.8%. The average differences for tensile energy absorption and Frag were the least precise--namely, 19.1 and 20.9%, respectively. Inspection of the distribution of differences in Table VI also reveals that the relationship involving Instron fatigue lives gave better predictions of sack performance. The same conclusion is arrived at when considering the regular and extensible papers separately in Table VI (the only major change is a marked improvement in the average per cent difference with tensile energy absorption for regular papers). Thus, in terms of the performance of the sacks of the second fabrication program at 50% R.H., the highest precision in predicting progressive height face drop performance was achieved with the Instron uniaxial fatigue life.

It may be remarked that the Instron fatigue life for a given sack sample was determined from only six specimens of paper corresponding to the outer ply of the sacks. The Frag and Thwing-Albert impact fatigue values in Table VI were obtained from 72 specimens representing all three plies of the sack (i.e., 24 per ply); tensile energy absorption was evaluated on 36 specimens. It is presumed that more extensive replication of the Instron fatigue tests and sampling from all three plies would not worsen the precision exhibited in the present study and in all likelihood would improve it.

Although the prediction of sack performance from Instron fatigue life was somewhat more precise than that based on conventional properties, it is probably not a great enough improvement to justify the laborious and time-consuming testing required to obtain fatigue life with an Instron testing machine (or other universal tension testing machine). It is visualized, however, that the merit of the Instron fatigue property lies not in the test method but rather in that uniaxial fatigue is amenable to mathematical description, as illustrated in References (2) and (3). It was demonstrated in those studies that the time-consuming fatigue test may be circumvented by testing the paper in simple tension tests and thereafter predicting the fatigue life by means of an equation stemming from the mathematical description of repeated tension.

Perhaps of even greater importance is the prospect that the effects of fiber and processing variables which are reflected in the simple tension tests may be projected to fatigue life by means of the fatigue life equation, thereby giving more basic insight into the relationship between papermaking and performance of the sack. Inasmuch as papermaking variables have long been studied in terms of the tension load-elongation curve, it would seem that there is a considerable body of experience in papermaking which can be translated into fatigue life performance without the necessity of performing life tests.

While conventional fatigue tests such as Thwing-Albert impact fatigue and Frag may be used to evaluate the effects of papermaking variables, they provide little further insight into the mechanisms involved. Thus, it is believed that the Instron uniaxial fatigue performance of sack paper may play a useful role in sack paper technology--at least as a research tool.

Returning to consideration of the correlation between Instron fatigue life and other paper properties, Table VII lists the correlation coefficients for fatigue life and other tension properties of the parent sack paper at 50% R.H. This table supplements Table XVII of Reference (6) by inclusion of fatigue life. In addition to the conventional tension properties derivable from the load-elongation curve (stretch and work) these tables list the "elastic" and "plastic" components of virgin TEA as illustrated in Fig. 1 of Reference (6).

TABLE VII

INTERCORRELATION OF INSTRON FATIGUE LIFE AND OTHER TENSION PROPERTIES

(Regular and extensible papers at 50% R.H.)

	Instron Fatigue, in	Instron Fatigue, cross	Stretch, in	Stretch, cross	T.E.A., in	T.E.A., cross	"Elastic" T.E.A., in	"Elastic" T.E.A., cross	"Plastic" T.E.A., in	"Plastic" T.E.A., cross
	Correlation Coefficient									
Instron Fatigue, in	1.000	0.776	0.998	0.792	0.989	0.574	0.329	-0.451	0.992	0.636
Instron Fatigue, cross		1.000	0.759	0.935	0.781	0.714	0.252	-0.335	0.785	0.759
Stretch, in			1.000	0.776	0.991	0.566	0.352	-0.444	0.993	0.627
Stretch, cross				1.000	0.799	0.808	0.160	-0.335	0.808	0.853
T.E.A., in					1.000	0.587	0.413	-0.492	0.998	0.654
T.E.A., cross						1.000	0.153	0.045	0.591	0.990
"Elastic" T.E.A., in							1.000	-0.339	0.361	0.201
"Elastic" T.E.A., cross								1.000	-0.483	-0.097
"Plastic" T.E.A., in									1.000	0.657
"Plastic" T.E.A., cross										1.000

It may be seen that each uniaxial fatigue life is highly correlated with the virgin stretch in the corresponding direction of the paper (namely, 0.988 and 0.935 for machine direction and cross direction, respectively). This result was not unexpected inasmuch as experience with the mathematical analysis of uniaxial fatigue life (obtained by the simpler constant elongation process) of References (2) and (3) suggested that virgin stretch is a dominant factor in the fatigue life equation.

Table VI reveals that Instron fatigue is also highly correlated with T.E.A. in the machine direction--explainable by the equally high correlation between stretch and T.E.A. in this direction. Cross-direction fatigue life was not as well correlated with cross-direction T.E.A. (correlation coefficient = 0.714) and this again parallels the comparably low correlation between cross-direction stretch and T.E.A. The correlations between fatigue life and "plastic" T.E.A. follow the virgin T.E.A. correlations in view of the high correlation between virgin T.E.A. and its "plastic" component. In summary, Instron fatigue life appears to be highly correlated with virgin stretch and also with T.E.A. (or its plastic component) when the latter is highly correlated with stretch, namely, in the machine direction.

The correlations noted above between fatigue life and machine-direction virgin tension properties are probably enhanced by the wide range of machine-direction properties in the fabrication program materials. Tables VIII and IX list intercorrelation coefficients for regular and extensible papers separately. It may be seen that the aforementioned correlations deteriorate substantially when each type of sack paper is considered separately. Most notably, the correlation between machine-direction fatigue life and virgin stretch decreases to 0.593 for regular paper (as compared with 0.988 of Table VI). The general worsening of the correlations in Tables VII and VIII may be attributed to the effect of the remaining

TABLE VIII

INTERCORRELATION OF INSTRON FATIGUE LIFE AND OTHER TENSILE PROPERTIES

(Regular papers at 50% R.H.)

	Instron Fatigue, in	Instron Fatigue, cross	Stretch, in	Stretch, cross	T.E.A., in	T.E.A., cross	"Elastic" T.E.A., in	"Elastic" T.E.A., cross	"Plastic" T.E.A., in	"Plastic" T.E.A., cross
Instron Fatigue, in	1.000	0.099	0.593	0.103	0.617	0.077	0.390	0.118	0.573	0.067
Instron Fatigue, cross		1.000	0.150	0.878	0.279	0.710	0.270	0.132	0.202	0.745
Stretch, in			1.000	0.250	0.856	0.343	0.232	0.114	0.982	0.353
Stretch, cross				1.000	0.170	0.810	0.096	0.133	0.280	0.852
T.E.A., in					1.000	0.297	0.684	0.109	0.898	0.305
T.E.A., cross						1.000	0.055	0.607	0.356	0.995
"Elastic" T.E.A., in							1.000	0.177	0.294	0.036
"Elastic" T.E.A., cross								1.000	0.037	0.520
"Plastic" T.E.A., in									1.000	0.378
"Plastic" T.E.A., cross										1.000

TABLE IX

INTERCORRELATION OF INSTRON FATIGUE LIFE AND OTHER TENSILE PROPERTIES

(Extensible papers at 50% R.H.)

	Instron Fatigue, in	Instron Fatigue, cross	Stretch, in	Stretch, cross	T.E.A., in	T.E.A., cross	"Elastic" T.E.A., in	"Elastic" T.E.A., cross	"Plastic" T.E.A., in	"Plastic" T.E.A., cross
Instron Fatigue, in	1.000	0.101	0.932	0.149	0.885	0.222	0.162	-0.161	0.919	0.259
Instron Fatigue, cross		1.000	0.116	0.767	0.159	0.378	-0.127	-0.052	0.193	0.400
Stretch, in			1.000	0.148	0.944	0.183	0.277	-0.127	0.963	0.213
Stretch, cross				1.000	0.240	0.652	-0.168	-0.036	0.287	0.680
T.E.A., in					1.000	0.244	0.472	-0.353	0.988	0.316
T.E.A., cross						1.000	-0.056	0.267	0.271	0.984
"Elastic" T.E.A., in							1.000	-0.434	0.327	0.022
"Elastic" T.E.A., cross								1.000	-0.302	0.093
"Plastic" T.E.A., in									1.000	0.335
"Plastic" T.E.A., cross										1.000

parameters of the tension load-elongation curve which govern fatigue life (2, 3) (namely, elastic and plastic slopes, reload slope and proportional limit strain).

Lastly, Fig. 7 and 8 show graphically the relationship between Instron fatigue life and Frag fatigue life in each direction of the paper. The Frag test results are given in terms of safe number of drops so as to be in units comparable to Instron fatigue life (in contrast to Frag burst energy which is proportional to the cube root of the drop number). Figures 7 and 8 indicate that the Instron and Frag fatigue lives are not well correlated; this result is perhaps partially attributable to the fact that the Instron test studied here involved a progressively increasing applied strain while the Frag test applies a constant energy to the specimen and is furthermore apparently not a purely uniaxial test.

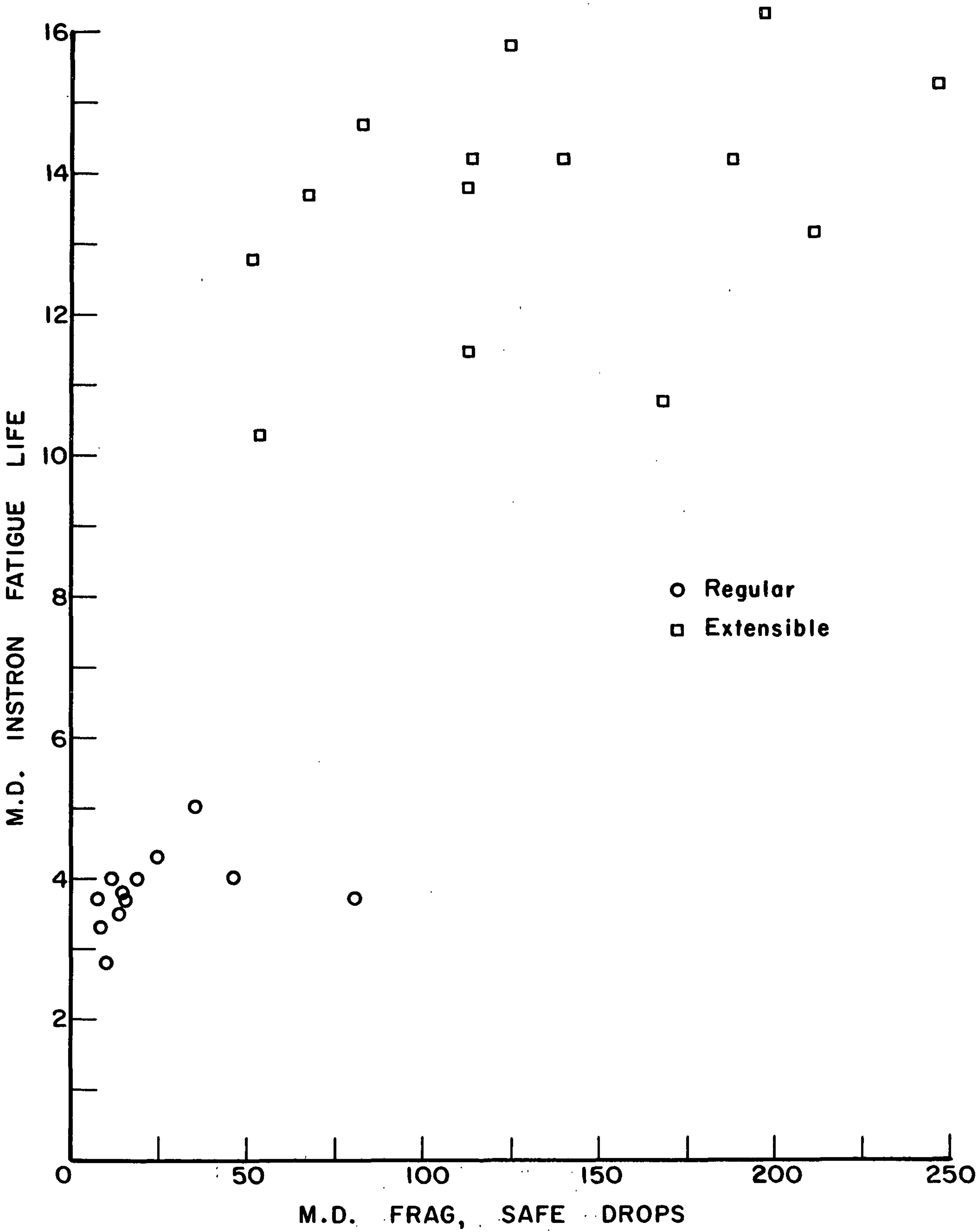

Figure 7. Relationship Between Machine-Direction Instron Fatigue Life and Machine-Direction Frag at 50% R.H.

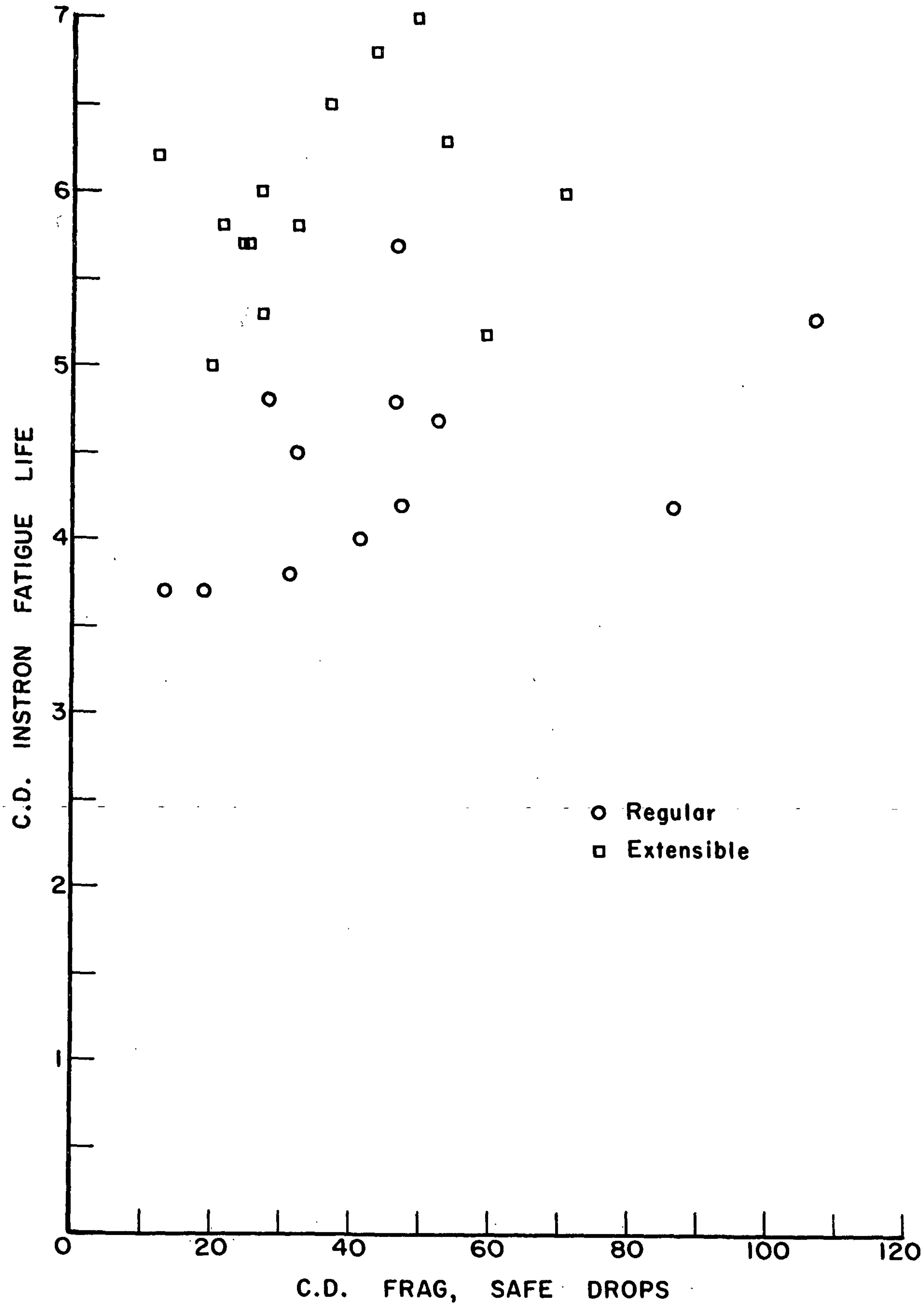

Figure 8. Relationship Between Cross-Direction Instron Fatigue Life and Cross-Direction Frag at 50% R.H.

PROPOSALS FOR FUTURE WORK

As mentioned earlier, the Instron fatigue life was evaluated on only six specimens from the outer ply of each sack sample. In the interest of more extensive sampling of the sack paper, it would be appropriate to continue the fatigue life tests on paper corresponding to the inner and middle plies of the sack. Moreover, if the replication were increased from six to twelve specimens per ply, the fatigue life data would be more nearly comparable in extent to that of the conventional paper properties of the second fabrication program, affording thereby a more reliable comparison of the predictive ability of the several types of paper properties.

It is believed, however, that prior to the above-mentioned continuation of the fatigue tests, it would be well to investigate whether other types of Instron repeated tension tests may be more appropriate to sack performance than the progressively increasing applied elongation schedule by which the papers were evaluated in the present study. Previous work has indicated that, with constant input schedules, applied elongation and applied energy processes are equivalent for regular papers but not for extensible paper. Thus, it is likely that samples of extensible paper may be ranked differently with an applied energy fatigue test than with an applied elongation test. Work has already been initiated with a progressively increasing applied <u>energy</u> process to determine whether it offers an improvement over applied elongation on the basis of the sacks tested at 50% R.H.

Following the mathematical analysis of Instron fatigue life that was presented in References (<u>2</u>) and (<u>3</u>), it was decided to hold in abeyance further analytical work until the worth of fatigue concepts in predicting sack performance could be evaluated. Now that it has been found that fatigue life ranks favorably with other paper properties, it may be appropriate to continue the mathematical analysis of the mechanics of repeated tension. Several avenues of further development were discussed in detail in References (2) and (3) involving variously (a) improvements in accuracy of the fatigue life equation, (b) extension of the analysis to various types of repeated tension processes and (c) simplification of the fatigue life equation. It would seem to be timely to extend the analyses to include applied energy processes and progressively increasing inputs. It is believed that work of the type proposed will both broaden and facilitate application of the concepts of fatigue to sack performance.

Finally, the present study emphasizes the need for a more basic understanding of the mechanics of sack impact. As mentioned earlier, the method of performing the Instron fatigue life tests was largely arbitrary, except that it bore some superficial resemblance to the progressive height sack impact test and endeavored to simulate the ratio of applied strains in both directions of the paper that had been observed in limited studies of sack impact. Fatigue life tests, unlike virgin property tests, are highly dependent on the nature and magnitude of the input process. Accordingly, it may be anticipated that the better relationships between fatigue-type tests and sack performance will proceed from those fatigue tests which more nearly simulate the stresses and strains induced in a sack. Relatively little is known at this time, however, about the behavior of the paper in the sack during impact. While there are probably many other compelling reasons for studying the mechanics of sack impact, it is believed that intensified effort in this area is warranted if only to further the application of fatigue concepts to sack performance.

LITERATURE CITED

1. The Institute of Paper Chemistry. Relationship between sack performance and the properties of the sack paper. Part I. Theoretical and experimental survey of the effect of fatigue (repeated applications of stress and/or strain) on the fundamental properties of paper. Project 2033, Progress Report Thirteen, March 23, 1960.

2. The Institute of Paper Chemistry. Relationship between sack performance and the properties of sack paper. Part II. A theory for the behavior of regular sack paper in repeated uniaxial tension. Project 2033, Progress Report Eighteen, July 25, 1961.

3. The Institute of Paper Chemistry. Relationship between sack performance and the properties of sack paper. Part III. Verification of a fatigue life theory for regular sack paper. Project 2033, Progress Report Nineteen, Oct. 9, 1961.

4. The Institute of Paper Chemistry. Investigation of the strains in a multiwall sack at the time of impact. Project 2033, Progress Report Nine, Sept. 8, 1959.

5. Unpublished work on progressively increasing elongation processes.

6. The Institute of Paper Chemistry. A study of multiwall sack performance. Part II. Comparative performance of sacks fabricated from regular and extensible sack papers and the relationship between sack performance and sack paper properties. Project 2033, Progress Report Twenty-One, Aug., 1962.

7. The Institute of Paper Chemistry. The I.P.C. line-type specimen clamp. Research Bull. 26, no. 2:56-9(Dec., 1959).

THE INSTITUTE OF PAPER CHEMISTRY

James W. Gander

James W. Gander, Research Aide
Container Section

R. C. McKee by wJ

R. C. McKee, Chief, Container Section

APPENDIX A

TABLE X

INSTRON FATIGUE LIFE AT 50% RELATIVE HUMIDITY

Fatigue Life, no. of strain applications

	Run AA In	Run AA Cross	Run BB In	Run BB Cross	Run CC In	Run CC Cross	Run DD In	Run DD Cross	Run EE In	Run EE Cross	Run FF In	Run FF Cross	Run GG In	Run GG Cross	Run HH In	Run HH Cross
Start	5	4	3	3	3	6	4	4	4	3	4	3	3	4	3	4
	5	5	3	4	4	6	4	4	4	3	4	4	4	5	3	4
	5	5	4	6	5	7	4	6	4	4	4	4	4	5	4	4
End	5	3	3	4	3	3	4	5	4	3	3	3	3	4	2	3
	5	4	4	5	4	6	4	5	4	4	3	4	4	4	2	4
	5	4	4	5	5	6	4	5	4	4	4	4	4	6	3	4
Av.	5.0	4.2	3.5	4.5	4.0	5.7	4.0	4.8	4.0	3.7	3.7	3.7	3.7	4.7	2.8	3.8

	Run II In	Run II Cross	Run JJ In	Run JJ Cross	Run KK In	Run KK Cross	Run LL In	Run LL Cross	Run MM In	Run MM Cross	Run NN In	Run NN Cross	Run OO In	Run OO Cross	Run PP In	Run PP Cross
Start	4	4	3	5	3	4	3	4	11	6	13	5	15	5	11	4
	4	4	4	6	3	4	3	5	11	6	14	6	16	5	11	5
	4	5	4	6	4	4	4	5	11	7	14	7	15	5	12	6
End	4	3	3	4	3	4	4	5	9	5	14	4	15	5	11	4
	5	4	4	5	3	4	4	5	11	5	15	6	15	6	12	6
	5	4	4	6	4	5	5	5	12	7	15	7	16	6	12	6
Av.	4.3	4.0	3.7	5.3	3.3	4.2	3.8	4.8	10.8	6.0	14.2	5.8	15.3	5.3	11.5	5.2

	Run QQ In	Run QQ Cross	Run RR In	Run RR Cross	Run SS In	Run SS Cross	Run TT In	Run TT Cross	Run UU In	Run UU Cross	Run VV In	Run VV Cross	Run WW In	Run WW Cross	Run XX In	Run XX Cross
Start	13	6	15	6	10	5	13	6	10	6	13	7	11	5	13	4
	14	6	16	8	10	6	13	6	16	6	13	7	14	6	14	4
	15	6	17	--	11	6	14	6	16	--	13	7	14	8	14	6
End	14	6	16	6	10	6	14	4	14	5	13	5	12	5	14	4
	14	7	17	7	10	6	14	6	16	6	13	7	13	6	14	6
	15	7	17	7	11	7	14	6	16	6	14	9	13	7	14	6
Av.	14.2	6.3	16.3	6.8	10.3	6.0	13.7	5.7	14.7	5.8	13.2	7.0	12.8	6.2	13.8	5.0

	Run YY In	Run YY Cross	Run ZZ In	Run ZZ Cross
Start	14	6	15	5
	14	6	15	6
	14	6	16	6
End	14	6	16	5
	14	7	16	5
	15	8	17	7
Av.	14.2	6.5	15.8	5.7

APPENDIX A (Continued)

TABLE XI

INSTRON FATIGUE LIFE AT 25% RELATIVE HUMIDITY

Fatigue Life, no. of strain applications

	In	Cross	In	Cross	In	Cross	In	Cross	In	Cross	In	Cross	In	Cross	In	Cross
	Run BB		Run EE		Run GG		Run II		Run JJ		Run KK		Run LL			
Start	3	3	3	2	3	3	4	3	3	2	3	3	3	4		
	3	4	3	3	3	3	4	3	4	4	4	4	4	4		
	4	5	3	4	3	4	4	4	4	4	4	4	4	4		
End	3	4	3	3	2	2	4	3	3	3	3	3	4	3		
	3	4	3	3	3	5	4	3	4	3	3	4	4	4		
	3	4	3	4	4	5	5	4	4	3	3	4	4	4		
Av.	3.2	4.0	3.0	3.2	3.0	3.7	4.2	3.3	3.7	3.2	3.3	3.7	3.8	3.8		
	Run MM		Run NN		Run OO		Run PP		Run QQ		Run RR		Run TT		Run WW	
Start	8	5	12	4	14	2	11	4	14	3	15	5	12	5	9	4
	10	5	12	4	15	4	11	5	14	5	17	5	13	5	13	5
	11	5	13	5	15	4	11	6	14	5	17	6	14	6	13	6
End	11	4	12	4	15	4	9	4	11	4	16	4	13	3	12	5
	11	4	12	4	15	4	11	5	13	5	16	4	13	5	12	6
	12	5	13	4	16	4	12	5	14	6	17	5	14	5	12	6
Av.	10.5	4.7	12.3	4.2	15.0	3.7	10.8	4.8	13.3	4.7	16.3	4.8	13.2	4.8	11.8	5.3

APPENDIX A (Continued)

TABLE XII

INSTRON FATIGUE LIFE AT 10% RELATIVE HUMIDITY

Fatigue Life, no. of strain applications

	In	Cross	In	Cross	In	Cross	In	Cross	In	Cross	In	Cross	In	Cross	In	Cross
	Run BB		Run EE		Run GG		Run II		Run JJ		Run KK		Run LL			
Start	3	2	3	2	3	2	3	3	3	3	3	2	3	2		
	3	3	3	3	3	3	4	3	3	4	3	3	3	3		
	3	3	3	3	3	4	4	3	4	4	3	3	3	3		
End	3	2	2	1	3	3	4	2	3	2	3	2	3	3		
	3	4	2	2	3	3	4	2	3	3	3	3	3	3		
	3	4	3	3	3	3	5	3	3	3	3	3	3	3		
Av.	3.0	3.0	2.7	2.3	3.0	3.0	4.0	2.7	3.2	3.2	3.0	2.7	3.0	2.8		
	Run MM		Run NN		Run OO		Run PP		Run QQ		Run RR		Run TT		Run WW	
Start	9	2	12	2	14	2	10	3	12	2	12	3	11	3	10	4
	9	3	12	3	14	2	10	4	13	5	14	4	11	4	11	4
	9	5	12	4	16	3	11	4	13	5	14	5	12	4	13	4
End	9	3	10	3	14	3	10	3	12	3	15	3	11	4	8	4
	10	3	11	3	15	4	11	3	12	3	15	4	11	4	10	5
	10	4	12	4	15	4	11	4	12	4	16	4	12	5	10	6
Av.	9.3	3.3	11.5	3.2	14.7	3.0	10.5	3.5	12.3	3.7	14.3	3.8	11.3	4.0	10.3	4.5